Interdisciplinary project in mathematics

Investigation of line-shaped and square-shaped discrete point structures: Grouping under different algorithms

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Summary

The groupings in discrete square structures and line structures are examined. The number of emerging groups is shown for both the sociality and polarity algorithm. There are two theorems for the number of groups with the polarity algorithms for line structures and square structures that I discovered.

Theorem 1:

Under the polarity algorithm, square structures of side length m, where m is straight, always form two groups of the same size $m^2/2$.

Theorem 2:

Under the polarity algorithm, for line structures, all even groups combine into a single group, all odd groups split into a single element and the rest of the group.

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CHAPTER ONE

Introduction

In this work, I examine two algorithms that I call sociality and polarity. I define sociality in such a way that every particle approaches the particle that is the only one closest to it. I define polarity in such a way that each particle approaches the particle that is the only one furthest away from it.

I analyze discrete structures, namely square lattice-shaped point structures and line-shaped point structures. I calculate how these structures group under the above algorithms. In doing so, I discover two theorems for the grouping of linear point structures under the polarity algorithm.

Definition polarity algorithm

The element merges with the element that is clearly identifiable as the element with the greatest distance.

Definition sociality algorithm

The element merges with the element that is clearly identifiable as the element with the smallest distance.

CHAPTER TWO
Square grid structures

Lattice structures with different numbers of points are examined.

2.1 Grouping of square grid structures under the sociality algorithm

Grid structure with 4 points

Sociality algorithm

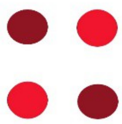

Image 2.1: Group size 2 (red).

Grid structure with 16 points

Sociality algorithm

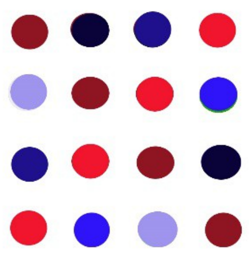

Image 2.2: Group sizes 4; 2 (red; blue).

In the event that the elements approach the next element, in a lattice structure this would mean that the closest element is the one that has the smallest spacing and for which no element with the same spacing exists. This would rearrange the lattice structure in such a way that two different types of particles are formed.

There are 2 particles with unit 4 and 4 particles with unit 2.

Grid structure with 32 points

Sociality algorithm

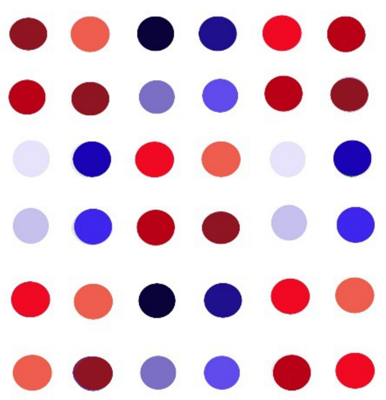

Image 2.3: Group sizes 5; 2 (red; blue).

Grid structure with 64 points

Sociality algorithm

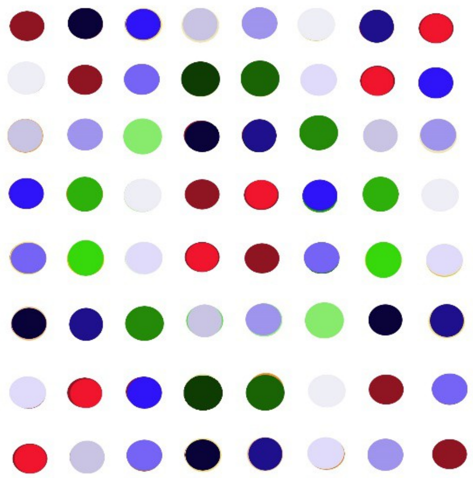

Image 2.4: Group sizes 6; 5; 2 (red; blue; green).

Grid structure with 100 points

Sociality algorithm

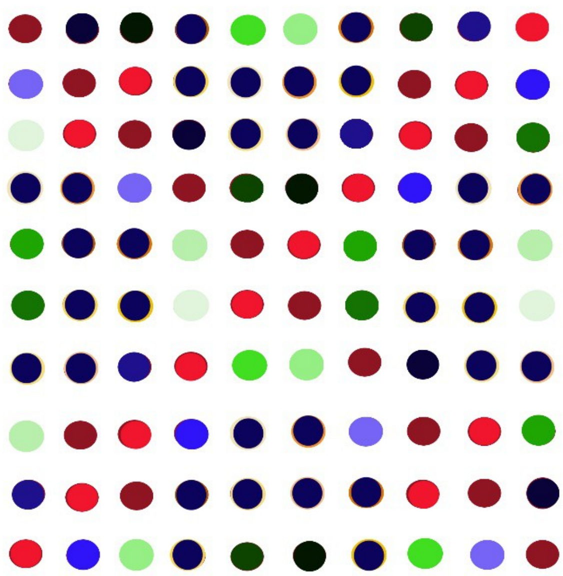

Image 2.5: Group sizes 14; 4; 3 (red; blue; green).

The group formation shown above is summarized here under sociality for square structures with two even side lengths n and m, where n=m, for n=m<= 10.

Number of points: 4 → 2 groups emerge.

(2)(2)

Number of points: 16 → 6 groups emerge.

(4)(4)(2)(2)(2)(2)

Number of points: 36 → 12 groups emerge.

(5)(5)(5)(5)(2)(2)(2)(2)(2)(2)(2)(2)

Number of points: 64 → 16 groups emerge.

(6)(6)(5)(5)(5)(5)(5)(5)(5)(5)(2)(2)(2)(2)(2)(2)

Number of points: 100 → 22 groups emerge.

(14)(14)(4)(4)(4)(4)(4)(4)(4)4)(4)(4)(4)(4)(3)(3)(3)(3)(3)(3)(3)(3)

Number of points	4	16	36	64	100
Number of emerging groups	2	6	12	16	22

Table 2.1: Number of groups in relation to the number of points in square structures.

2.2 Grouping of square lattice structures under the polarity algorithm

Theorem 1:

Under the polarity algorithm, square structures of side length m, where m is straight, always form two groups of the same size m²/2.

Image 2.6: Group sizes 8; 8 (red; blue).

The group formation under the polarity algorithm for square structures with two even side lengths n and m is summarized as follows, where n=m.

All such square structures form two groups of the same size (n*m)/2. The sketch with n=4, m=4 illustrates the arrangement of the groups.

CHAPTER THREE
Line structures

3.1 Grouping of line structures under the sociality algorithm

From 1 to 20, line structures with the following numbers of particles unite to form a single group through the sociality algorithm:

1, 2, 3, 4, 7, 10, 12, 15, 18.

Line structures with the following length combine to form two groups:

5, 6, 8, 9, 11, 16, 19.

Line structures with the following length combine to form three groups:

13, 17.

Line structures with the following length combine to form four groups:

14.

Line structures with the following length combine to form five groups:

20.

In the following, the line structures with 1-20 elements are shown under association based on the sociality algorithm.

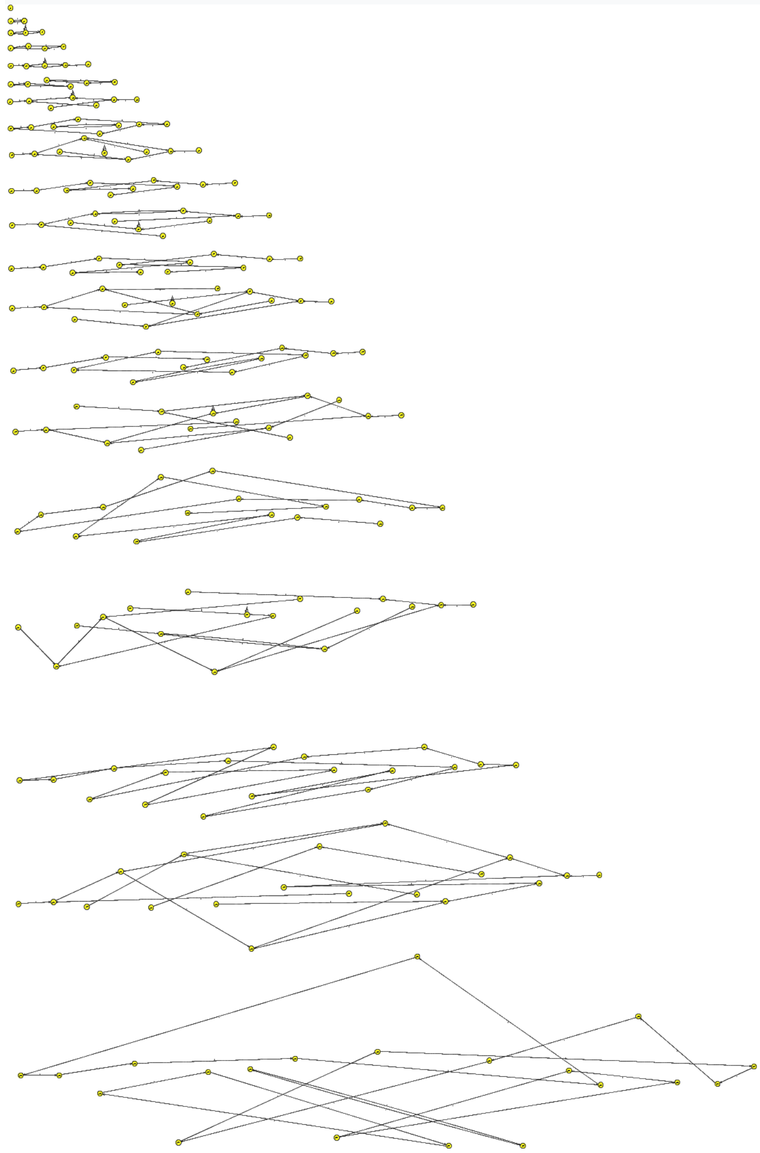

Image 3.1: Line structures from one to twenty based on the sociality algorithm.

The forms that arise in line structures from one to ten under the sociality algorithm are shown in the following picture:

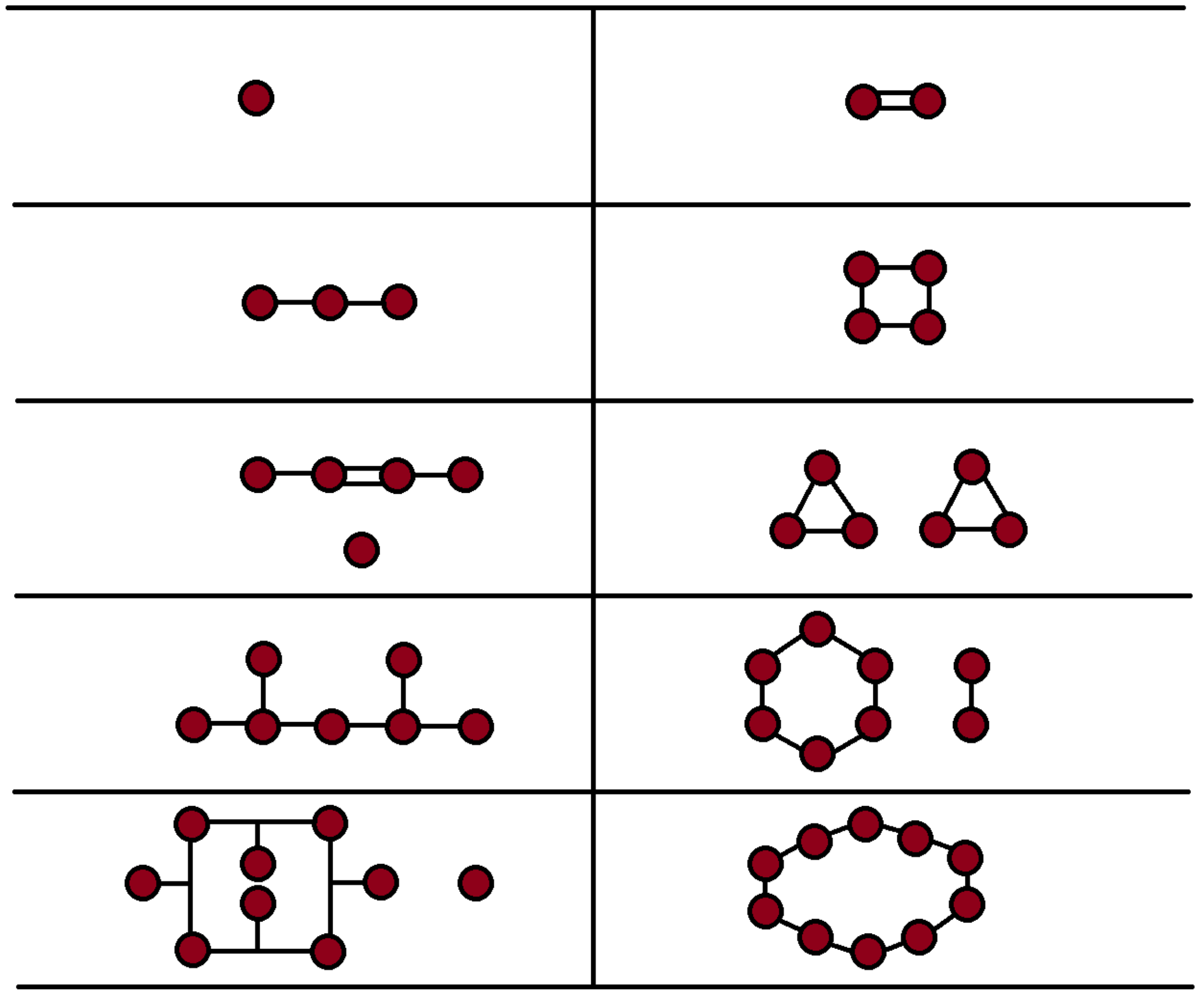

Image 3.2: Forms of line structures from one to ten based on the sociality algorithm

3.2 Grouping of line structures under the polarity algorithm

Theorem 2:

Under the polarity algorithm, all groups of even size combine into a single group, all groups of odd size split into a single element and the rest of the group.

In the following, the line structures with 2-11 elements are shown under connection based on the polarity algorithm.

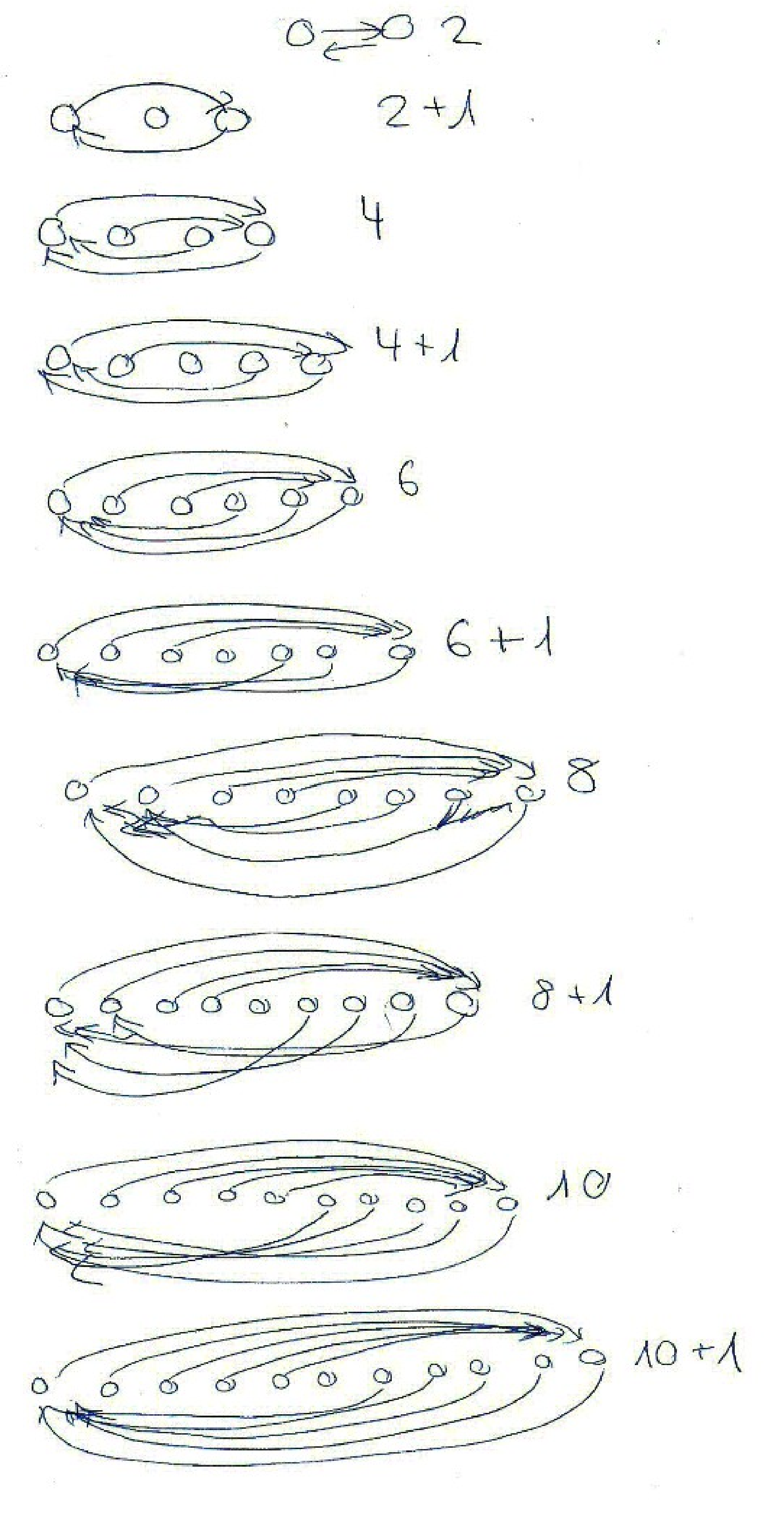

Image 3.3: Line structures from two to eleven based on the polarity algorithm

CHAPTER FOUR

Conclusion

The function that maps the natural numbers to the number of resulting groups under the sociality algorithm with linear point structures initially (with the first 1000 numbers) still contains more than half (51.6%) prime numbers, but becomes more and more balanced with larger numbers. That is, the percentage of prime numbers decreases. For the natural numbers up to 3000, the percentage is only 46.57%.

Again and again a single group emerges, but sometimes several hundred of groups.

LIST OF FIGURES

LIST OF TABLES